

where  $\varphi$  and  $\theta$  are positive parameters.<sup>8</sup>

At a point in time the money wage is given, changing over time in line with the gap between the wage share desired by workers,  $\sigma_w$ , and the actual wage share:

$$\hat{W} = \mu[\sigma_w - \sigma] \quad (10)$$

where  $\hat{W}$  is the proportionate rate of change in money wage, and  $0 < \mu \leq 1$  is the speed of adjustment.<sup>9</sup> The wage share desired by workers is assumed to depend on workers' bargaining power, which rises with the employment rate:

$$\sigma_w = \lambda e \quad (11)$$

where  $\lambda$  is a positive parameter and  $e$  is the employment rate,  $L / N$ , which can be linked to the state of the goods market in the following way

$$e = auk \quad (12)$$

where  $k$  stands for the ratio of capital stock to labor supply, that is,  $k = K / N$ , with  $N$  being the supply of labor. This formal link between  $u$  and  $e$  is necessary because the fixed-coefficient nature of the technology implies that an increase in output in the short run will necessarily be accompanied by an increase in employment.

Since capital accumulation is demand-driven, the macro equality between investment and saving will be brought about by changes in output through changes in capacity utilization. Assuming that capital does not depreciate,  $g$ , the rate of capital accumulation, which is the growth rate for this one-good economy, is

$$g = r \quad (13)$$

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(8) Wood (1975) and Eichner (1976) argue that during expansions firms may want to invest more by generating higher internal savings and therefore desire a higher markup. Rowthorn (1977) suggests that higher levels of capacity utilization allows firms to raise prices with less fear of being undercut by their competitors, who would gain little by undercutting due to higher capacity constraints. Gordon, Weisskopf & Bowles (1984) argues that marked-up prices are inversely related to the perceived elasticity of demand, which is a negative function of both the degree of industry concentration and of the fraction of the firm's potential competitors who are perceived to be quantity-constrained and hence not engaged in or responsive to price competition. The conclusion is that in the downturn the markup will fall because the general fall in capacity utilization gives rise to a smaller share of the firm's potential competitors being perceived to be operating under capacity constraints, and hence to an increase in the perceived elasticity of demand facing the firm.

(9) One of the reasons why Keynes (1936) rejected the classical second postulate is that it flows from the mistaken idea that the wage bargaining determines the real wage. For Keynes, the postulate that there is a tendency for the real wage to come to equality with the marginal disutility of labor incorrectly presumes that labor is in a position to decide the real wage for which it works, even though not the quantity of employment forthcoming at this wage. Keynes' economics of employment is analyzed in detail in Lima (1992).

given the assumptions that workers do not save and capitalists save all of their income.

## 2 The behavior of the model in the short run

The short run is defined as a time frame in which the capital stock,  $K$ , the labor supply,  $N$ , the price level,  $P$ , and the money wage,  $W$ , can all be taken as given. The existence of excess capacity implies that output will adjust to remove any excess demand or supply, so that in short-run equilibrium,  $g = g^d$ . Substituting from (3), (6), and (13), we can solve for the equilibrium value of  $u$ , given  $\sigma$  and the other parameters:

$$u^* = \frac{\alpha + \gamma\sigma - \delta\sigma^2}{(1 - \sigma) - \beta} \quad (14)$$

Meaningful values for the wage and profit shares are required, and a positive profit share is automatically ensured by  $z > 0$ . A positive wage share is ensured by  $z < +\infty$ , which we assume. As for stability, it is assumed a Keynesian short-run adjustment mechanism stating that output will change in proportion to the excess demand in the goods market, which means that the short-run equilibrium value for  $u$  will be stable provided the denominator of the expression in (14) is positive. This is ensured by the usual condition for macro stability that aggregate saving is more responsive than investment to changes in output (capacity utilization), which here implies that  $\beta$  has to be low, a condition we assume to be satisfied. Since  $\alpha$  is positive, the numerator of the expression for  $u^*$  will be positive throughout its economically relevant domain, given by  $0 < \sigma < 1$ .

A natural issue to address regards the impact of changes in the wage share on capacity utilization. However, whether or not a higher (lower) wage (profit) share will increase the degree of capacity utilization is ambiguous, given that while aggregate savings,  $g$ , is, given  $u$ , linear in the wage share, firms' desired accumulation plans is non-linear in distribution. Now, both aggregate savings and desired accumulation are functions of capacity utilization as well, and to avoid that  $u^*$  becomes unstable in the short run we assumed that the former is more

responsive than the latter to changes in  $u$ . All this ambiguity is captured by the formal expression for  $u_{\sigma}^*$ :

$$u_{\sigma}^* = \frac{(\gamma - 2\delta\sigma) + u}{(1 - \sigma) - \beta} \quad (15)$$

The restrictions already placed in the parameters ensure that  $u$  is positive throughout its (economically) meaningful domain. Hence, for the numerator in (15) to become negative, making for  $u_{\sigma}^* < 0$ , a rise (fall) in the wage (profit) share from a given level has not only to reduce firms' desired investment, but also to do so by more than it reduces saving. Now, the intensity of the response of aggregate savings to a change in the wage share depends on the level of capacity utilization, which depends on the level of wage share itself. Substituting (14) into (15), we have

$$u_{\sigma}^* = \frac{\delta\sigma^2 - 2\delta(1 - \beta)\sigma + \gamma(1 - \beta) + \alpha}{[(1 - \sigma) - \beta]^2} \quad (16)$$

The numerator of this expression for  $u_{\sigma}^*$  is a concave-up parabola, and further convenient restrictions in the parameters will ensure that it will be zero at some  $\sigma^* < \sigma^+ < 1$  and at some  $\sigma \geq \gamma / \delta$ , with  $\sigma^* = \gamma / 2\delta$  being the wage share at which desired accumulation, given  $u$ , is the highest. These will be the levels at which the expression for  $u_{\sigma}^*$  will change sign, and given that its numerator is a concave-up parabola,  $u_{\sigma}^*$  will be positive for  $\sigma < \sigma^+$ , and negative for  $\sigma > \sigma^+$ . While for low, intermediate-low and intermediate-high levels of wage share a higher (lower) wage (profit) share will increase capacity utilization, the reverse will happen for high levels of the wage share. Borrowing the terminology employed by Bhaduri & Marglin (1990), an stagnationist capacity utilization regime prevails at low, intermediate-low and intermediate-high levels of wage share, whereas an exhilarationist one prevails at high levels of wage share.<sup>10</sup> When  $\sigma < \sigma^*$ , a higher

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(10) As recalled above, Bhaduri & Marglin (1990) maintains that it is theoretically unsound to make investment to depend on capacity utilization and the rate of profit, since it is not certain that an increase in capacity utilization will induce additional investment when the profit rate is held constant -- the reason being that in case capacity utilization increases while the rate of profit remains constant, it must be the case that the profit share falls. Since Bhaduri & Marglin (1990) uses an investment function which is implicitly positively linear in the profit share, the way it allows for the possibility that capacity utilization may rise (fall) when the profit shares rises is by suggesting that investment may be more (less) responsive than savings to changes in the profit share at high (low) levels of capacity utilization. Hence, while stagnation or wage-led capacity utilization may be more likely at low levels of capacity utilization, exhilarationism or profit-led capacity utilization may be more likely at high levels of capacity utilization. In the model of this paper, the desired investment function, by being explicitly non-linear in distributive shares, implies more naturally that whether investment or saving is more responsive to changes in profitability depends on the level of profitability.

wage share will unambiguously increase capacity utilization, as in the newer post keynesian model developed independently by Rowthorn (1981) and Dutt (1984, 1990). While in these models the rise in  $u$  is due to the corresponding rise in consumption demand, in the model of this paper a positive profitability effect will add to that mechanism, in that firms' accumulation plans are rising even though the profit share is falling. When  $\sigma > \sigma^*$ , the positive impact of a higher wage share on consumption demand is accompanied by a negative impact of lower profitability on desired investment. Since the intensity of this negative impact rises with the gap between  $\sigma^*$  and  $\sigma$ , the chances for  $u_\sigma^* < 0$  will be the higher, the closer  $\sigma$  gets to the upper bound for the wage share. Hence, suitable restrictions in the parameters will ensure that the numerator of (16) will be zero at  $\sigma^* < \sigma^+ < 1$  and at some  $\sigma \geq \gamma / \delta$ , so that a rise in the wage share will rise capacity utilization for some intermediate-high levels of wage share as well. For high levels of wage share, though, the squeeze in profits brought about by a higher wage share will be strong enough to make for  $u_\sigma^* < 0$ .

Given our assumptions that workers do not save and capitalists save all of their income, the rates of profit and accumulation will be the same, and how these rates will respond to changes in distribution is something that becomes likewise ambiguous. Given capacity utilization, a rise (fall) in the wage (profit) share will unambiguously exert a downward pressure on the accumulation rate. However, the non-linear nature of the desired accumulation function makes for the possibility that a higher wage share generates a rise in capacity utilization which may eventually more than compensate the accompanying fall in the profit share. For high levels of wage share ( $\sigma > \sigma^+$ ), a redistribution towards workers will unambiguously slow down the rate of accumulation, the reason being that the resulting profit squeeze will reduce capacity utilization via its negative impact on desired accumulation. For the remainder of the relevant domain ( $\sigma < \sigma^*$ ), a rise in the wage share, by raising capacity utilization, may eventually speed up the accumulation rate, with the resulting profit-squeeze not being deleterious to capital accumulation. Hence, the non-linear desired accumulation function assumed here allows us to derive some precise distributional conditions under which the relationship between distribution and accumulation is either positive or negative. All this ambiguity is captured by the formal expressions for  $g^*$  and  $g_\sigma^*$ , which, using equations (6), (13), (14), and (16), are

$$g^* = \frac{\delta\sigma^3 - (\gamma + \delta)\sigma^2 + (\gamma - \alpha)\sigma + \alpha}{(1 - \sigma) - \beta} \quad (17)$$

$$g_{\sigma}^* = \frac{-2\delta\sigma^3 + [3\delta(1-\beta) + (\gamma + \delta)]\sigma^2 - 2(1-\beta)(\gamma + \delta)\sigma + (1-\beta)\gamma + \alpha\beta}{[(1-\sigma) - \beta]^2} \quad (18)$$

Further suitable restrictions in the parameters will ensure that the cubic expression in the numerator of (18) be zero at some  $\sigma < 0$ , at  $\sigma = \sigma^*$ , and at some  $\sigma \geq \gamma / \delta$ . For low and intermediate-low levels of wage share ( $\sigma < \sigma^*$ ) wage-led growth will prevail, whereas for intermediate-high and high levels of wage share ( $\sigma > \sigma^*$ ) it is profit-led accumulation which will obtain. Therefore, the meaningful subset of the domain can be divided into three regions. In the first one, comprised by low and intermediate-low levels of wage share ( $\sigma < \sigma^*$ ), capacity utilization and accumulation are both directly related to the wage share. We refer to this region as LW in what follows. In the second region, which comprises intermediate-high levels of wage share ( $\sigma^* < \sigma < \sigma^+$ ), even though capacity utilization is directly related to the wage share, changes in the profit share are assumed to dominate changes in capacity utilization. Within this region, therefore, referred to as IH, accumulation is inversely related to the wage share. In the third region ( $\sigma > \sigma^+$ ), in turn, to which we refer as HW, capacity utilization and accumulation are both inversely related to the wage share.

### 3 The behavior of the model in the long run

In the long run we assume that the short-run equilibrium values of the variables are always attained, with the economy moving over time due to changes in the stock of capital,  $K$ , the supply of labor,  $N$ , the price level,  $P$ , and the money wage,  $W$ . One way of following the behavior of the system over time is by examining the dynamic behavior of the short-run state variables  $\sigma$ , the wage share, and  $k$ , the ratio of capital stock to labor supply, and this is the analytical alternative pursued here. From the definition of these variables, and denoting time-rates of change by overhats, the state transition functions are:

$$\hat{\sigma} = \hat{W} - \hat{P} + \hat{a} \quad (19)$$

$$\hat{k} = \hat{K} - \hat{N} \quad (20)$$

Substitution from (11) and (12) into (10), and from the resulting expression into (19), along with substitution from (7) into (9), and from the resulting expression into (19), will yield

$$\hat{\sigma} = \mu(\lambda auk - \sigma) - \tau(\sigma - \varphi + \theta u) \quad (21)$$

where  $u$  is given by equation (14). Since we abstract from technological change, the labor-output ratio is assumed to remain unchanged throughout, which implies that the dynamics of the wage share is governed by the dynamics of the real wage.

Now, substituting from (6) into (13) and then the resulting expression into (20), we obtain

$$\hat{k} = (1 - \sigma)u - n \quad (22)$$

where  $u$  is given by equation (14), while  $n$  is the growth rate of labor supply. The latter is here assumed to be exogenously given, but later we take it as being endogenously given.

Equations (21) and (22), after using (14), constitute a planar autonomous two-dimensional system of non-linear differential equations in which the rates of change of  $\sigma$  and  $k$  depend on the levels of  $\sigma$  and  $k$ , and on the level of parameters. The matrix  $M$  of partial derivatives for this dynamic system is given by

$$M_{11} = \partial \hat{\sigma} / \partial \sigma = \mu(\lambda a k u_{\sigma}^* - 1) - \tau(1 + \theta u_{\sigma}^*) \quad (23)$$

$$M_{12} = \partial \hat{\sigma} / \partial k = \mu \lambda a u^* > 0 \quad (24)$$

$$M_{21} = \partial \hat{k} / \partial \sigma = g_{\sigma}^* \quad (25)$$

$$M_{22} = \partial \hat{k} / \partial k = 0 \quad (26)$$

Eq. (24) shows that an increase in the capital-labor supply ratio, by raising the employment rate, will raise the wage share desired by workers,  $\sigma_w$ , which will raise the rate of money wage increase. Eq. (26) shows that since an increase in  $k$  does not affect either  $\sigma$  or  $u$ , there is no effect on the rate of accumulation, and hence no effect on the rate of growth of  $k$ .

Let us now turn to those partial derivatives whose signs are ambiguous, an ambiguity mainly associated with the non-linearity of the desired investment function. Eq. (23) shows that the impact of a change in the wage share on its own rate of change is mediated by the accompanying impact on capacity utilization. The reason is that the wage share desired by workers and the wage share implied by firms' desired markup depend on capacity utilization. While  $\sigma_f$  depends directly on the state of the goods market,  $\sigma_w$  depends directly on the state of the labor market. Now, given the fixed-coefficient nature of the assumed production

technology, an increase in capacity utilization in the short run will necessarily be accompanied by an increase in employment. Therefore, the sign of this partial derivative will depend on the relative strength of the two effects. As for the sign of  $\partial \hat{k} / \partial \sigma$ , (26) shows that it is governed by the impact of changes in the wage share on the rate of accumulation.

We now have all the elements for a qualitative phase-diagrammatic analysis of the (local) stability properties of this dynamic system. The way we proceed is by analyzing the stability of an equilibrium position in each one of the three regions into which we divided the relevant domain. Now, (18) shows that the numerator of  $g = (1 - \sigma)u$  is cubic in the wage share, while its denominator is linear in the wage share. With  $n$  being exogenously given, the equation describing the  $\hat{k} = 0$  isocline is, therefore, cubic in the wage share as well, which means that there may be up to three real values for the wage share in the  $(k - \sigma)$ -space at which a corresponding vertical  $\hat{k} = 0$  isocline will be located. Given this geometry, we proceed by analyzing the stability of the equilibrium position in each one of those regions were one of the  $\hat{k} = 0$  isoclines to be located there.

In the LW region ( $\sigma < \sigma^*$ ), capacity utilization and accumulation are directly related to the wage share. Hence, a higher wage share will exert an upward pressure on its own rate of change by raising capacity utilization and thus employment, which will raise the wage share desired by workers. However, this same rise in capacity utilization will also raise the markup desired by firms, which will then exert a downward pressure on the rate of change of the wage share by lowering the wage share ‘desired’ by firms. The sign of  $M_{11}$  will then depend on the relative strength of these two effects, which in turn is governed by the parameters  $\lambda$  and  $\theta$ , and by the speeds of adjustment  $\mu$  and  $\tau$ , respectively. Besides, the growth rate is directly related to the wage share within this region, which means that the rate of growth of  $k$  will rise with the wage share to make for  $M_{21} > 0$ . Since the level of  $k$  influences the strength of the effect of a change on capacity utilization on the rate of change of nominal wages – given that  $e = auk$  – it is likely that the chances for  $M_{11} > 0$  will be higher, the closer to  $\sigma^*$  the  $\hat{k} = 0$  isocline – and thus the equilibrium position – is located in the subset of the domain defined by this region. In any case, nothing can be ruled out in terms of the sign of  $M_{11}$ .

In case  $M_{11} < 0$ , the resulting steady-state solution will be a saddle-point. Indeed, the resulting steady-state will be a saddle-point anyway, the reason being

that  $\text{Det}(M) < 0$  no matter the sign of  $M_{11}$ . Hence, the resulting steady-state solution will not be stable in the subset of the relevant domain in which wage-led accumulation ( $g_{\sigma}^* > 0$ ) obtains.

In the IH region ( $\sigma^* < \sigma < \sigma^+$ ), capacity utilization is still directly related to the wage share. Changes in the profit share are assumed to dominate changes in capacity utilization, though, thus implying that the accumulation rate is inversely related to the wage share. Therefore, a higher wage share will exert an upward pressure on its own rate of change by raising capacity utilization and thus employment, which will then raise the wage share desired by workers. But as in the previous region, this same rise in capacity utilization will simultaneously raise firms' desired markup, which will then put a downward pressure on the rate of change of the wage share by lowering firms' 'desired' wage share. In principle, therefore, the sign of  $\partial \hat{\sigma} / \partial \sigma$  is as ambiguous as in the LW region. Since the growth rate is inversely related to the wage share now, thus making for  $M_{21} < 0$ ,  $\text{Det}(M)$  now becomes positive. With the possibility of a saddle-point thus ruled out, whether the equilibrium solution will be stable or unstable depends on whether  $M_{11}$  is negative or positive, respectively.

In the HW region ( $\sigma > \sigma^+$ ), capacity utilization and accumulation are now inversely related to the wage share. As in the IH region, the accumulation rate being inversely related to the share of wages in income makes for  $\text{Det}(M) > 0$ , which in turn rules out the possibility of a saddle-point equilibrium. Whether the steady-state solution will be stable or unstable depends therefore on whether  $\text{Tr}(M)$  is negative or positive. Now, the sign of  $\text{Tr}(M)$  is given by the sign of  $M_{11}$ , which is again ambiguous. A higher wage share will put a downward pressure on its own rate of change by lowering capacity utilization and thus employment, which will lower the wage share desired by workers. However, this same fall in capacity utilization will lower firms' desired markup as well, which will then put a upward pressure on the rate of change of the wage share by lowering the rate of change in prices. Therefore, the sign of  $\partial \hat{\sigma} / \partial \sigma$  depends on the relative strength of these two effects, it being positive (negative) in case the fall in capacity utilization brought about by a rise in the wage share puts a stronger pressure on the rate of change of prices (nominal wages). Even though the effect through the rate of change in nominal wages might be expected to be strong at such high (low) levels of wage (profit) share, the possibility for a positive sign for  $\partial \hat{\sigma} / \partial \sigma$  should not be ruled out.



The relative strength of these two effects will thus have different stability implications in the IH ( $\sigma^* < \sigma < \sigma^+$ ) and HW ( $\sigma > \sigma^+$ ) regions. In the IH region, stability (instability) obtains when the rate of change in prices is more (less) responsive than the rate of change in nominal wages to a change in capacity utilization generated by a change in the wage share. In the HW region, in turn, stability (instability) obtains when the rate of change in nominal wages is more (less) responsive. The reason for this required – for stability purposes – inversion in the relative strength of the two effects is that capacity utilization and employment are directly (inversely) related to the wage share in the IH (HW) region. More precisely, in case a rise in the wage share generates a higher (lower) capacity utilization the stability requirement is that the latter causes the rate of change in the nominal wages to rise by less (more) than the rate of change in prices. In other words, while in the LW region ( $\sigma < \sigma^*$ ) the prevalence of wage-led growth prevents stability completely, in the subset defined by intermediate-high (IH) and high (HW) levels of wage share the stability requirement is that the nominal wage change effect is weak (strong) when wage-led (profit-led) capacity utilization prevails.

#### 4 Long-run behavior with endogenous labor supply growth

The foregoing dynamic analysis was developed under the assumption that labor supply grows at an exogenously given rate. We now modify the autonomous two-dimensional system of non-linear differential equations given by (21) and (22) to endogeneize the growth rate of labor supply, while keeping intact the other building blocks of the model. Since a constant rate of unemployment as a long-run characteristic implies growth of employment equal to growth of the labor force, there are at least two possibilities to be considered here. The first is that firms' desired accumulation plans somehow conform to an exogenously given growth of the labor force, as in the previous dynamic analysis.<sup>11</sup> The second is the flexibility of the growth of the labor supply through variations in the average age of people joining and leaving the labor force, movements into or out of the household, and

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(11) Which does not mean, however, that the model is a supply-constrained model in the usual sense. First, the economy still has unemployed labor at long-run equilibrium. The rate of employment has stabilized to stabilize the wage share, which is required for long-run equilibrium. Second, unlike models with exogenously-given growth, this model has examined the dynamics of the economy out of long-run equilibrium in which it is driven by demand as well as supply forces. Since the economy will arrive, if at all, at long-run equilibrium only in the limit, and since exogenous parametric shifts may be shifting this equilibrium frequently, the economy is not usually supply-constrained in the sense that it always grows at the rate of labour supply growth.

migration. Thus, at least over some range, the growth of the labor force is able to adjust to the growth of the capital stock.<sup>12</sup>

The way we endogeneize  $n$  is by making it to depend on the dynamics of the labor market, and a reasonable assumption is to make it to depend on the level of employment:

$$n = \xi + \psi e \quad (27)$$

where  $\xi$  represents an autonomous component,  $\psi$  is a positive parameter, and  $e$  is the employment rate given by (12). The introduction of an endogenous mechanism of labor supply growth allows the labor supply to adjust, at least over some range, to meet labor demand in the long run. Hence, the long-run equilibrium is determined mostly by capital accumulation: to some extent, the natural rate of growth adapts to the warranted rate. A possible rationale for this specification is that the tighter the labor market, the higher the prospects of finding – and keeping – a position for those outside the labor force, which will then animate them to (re)join the labor force at the prevailing nominal wage. In other words, this specification is more inclusive than the preceding one in the sense that it incorporates a feedback effect from the dynamics of the goods and labor markets to the dynamics of the labor force. Now a higher rate of employment will not only induce those already employed to conduct nominal wage bargain in a way that ensures a (potentially) higher wage share, but will also stimulate those outside the labor force to (re)join it.

As a matter of fact, Velupillai (1993) develops an interesting growth model along classical lines by re-specifying Johansen's (1967) famous formulation of the classical model. In Velupillai's re-specification, the growth rate of labor supply is positively related to the rate of growth of the money wage and the share of wages in income. With the growth rate of money wages being positively related to the employment rate, the growth rate of labor supply becomes positively related to the

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(12) According to Sawyer (1989), the supply of labor to the capitalist economies (and within capitalist economies, supply to the capitalist sectors) can, at least over some range, be readily expanded whenever it is necessary. Within a country, the capitalist economy may cover only a part of the economy, so that the capitalist sector can pull workers from the non-capitalist sector when demand for labor is relatively high and push workers back when demand is low. Other mechanisms include migration of labor from one country to another and changes in the age of entry into and departure from the labor force. Therefore, extra supply of labor can be obtained when demand is strong by pulling people into the labor force. Conversely, when the overall demand for labor is low, unemployment can to some degree be hidden by the re-absorption of workers back into home.

employment rate and the wage share. In Velupillai's view, these equations derive from the most classical assumption of the model by Johansen: growth of labor supply as a function of the difference between what Ricardo called the market price of labor (wage rate in terms of goods) and the natural price of labor (wage rate that allows workers to subsist and perpetuate, without increase or diminution). The market price, in turn, was determined as a function of the natural price and the rate of employment, where the latter measures the momentary scarcity of labor and gives the supply price of labor.<sup>13</sup>

For Velupillai, these assumptions are actually mathematically equivalent to assuming a given bargained money wage determining a lower limit and the growth of the market rate being, then, a function of the employment rate. In the model of this essay, which introduces explicitly price dynamics through a conflict theory of inflation, the growth rate of nominal wages is positively related to the employment rate through an endogenously determined desired wage share. Capacity utilization and the rate of accumulation being variable, the rate of employment depends non-linearly on the wage share. Hence, it seems reasonable to make the growth rate of labor supply to depend on the employment rate only.

Having been modified in this manner, our two-dimensional non-linear dynamic system is now given by

$$\hat{\sigma} = \mu(\lambda auk - \sigma) - \tau(\sigma - \varphi + \theta u) \quad (21)$$

$$\hat{k} = (1 - \sigma)u - \xi - \psi auk \quad (28)$$

where  $u$  is given by (14). Again, we have an autonomous two-dimensional system in which the rates of change of the state variables  $\sigma$  and  $k$  depend on the levels of  $\sigma$  and  $k$ , and on the level of parameters. The matrix  $M^+$  of partial derivatives for this system is:

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(13) Formally, Johansen assumes that employed labor,  $L$ , is proportional to the capital stock,  $K$ , which means that the labor to capital ratio being constant, labour demand is determined by the capital stock. The labor supply function can be written as  $L = Ng(w - w^*)$ , where  $N$  is the total population,  $w$  is the market wage rate,  $w^*$  is the natural wage rate, and the function  $g$  is the per capita labour supply function, the latter being increasing in the wage rate. The previous expression can be inverted to yield the supply price of labor, that is,  $w = w^* + G(L/N)$ , with  $G$  standing for the inverse of the function  $g$ . In the longer run labor supply is influenced by population growth in a way given by  $dN/dt = Nh(w - w^*)$ . When  $w = w^*$ ,  $dN/dt = 0$ , which may be interpreted as an implicit definition of the natural wage rate. As we saw above, Ricardo defined the natural price of labor as the one that allows workers to subsist and perpetuate, without increase or decline.

$$M_{11}^+ = \partial \hat{\sigma} / \partial \sigma = \mu(\lambda a k u_{\sigma}^* - 1) - \tau(1 + \theta u_{\sigma}^*) \quad (29)$$

$$M_{12}^+ = \partial \hat{\sigma} / \partial k = \mu \lambda a u^* > 0 \quad (30)$$

$$M_{21}^+ = \partial \hat{k} / \partial \sigma = g_{\sigma}^* - \psi a k u_{\sigma}^* \quad (31)$$

$$M_{22}^+ = \partial \hat{k} / \partial k = -\psi a u^* < 0 \quad (32)$$

Not all the partial derivatives for this dynamic system can be unambiguously signed. As the state transition function for the wage share given by (21) has not changed, the sign of  $\partial \hat{\sigma} / \partial \sigma$  is as ambiguous as in the preceding system, while the sign of  $\partial \hat{\sigma} / \partial k$  is likewise positive. Since the rate of change in  $\hat{k}$  was now made to depend on the level of  $k$  as well, the other partial derivatives will change with respect to the foregoing analysis. Since we have endogeneized the rate of growth of labor supply, the sign of  $\partial \hat{k} / \partial \sigma$  becomes more ambiguous. A change in the wage share, by changing capacity utilization, will now affect both the rate of growth of the capital stock and the rate of growth of labor supply, and we analyze this ambiguity for each region of the domain in detail below. As for the sign of  $\partial \hat{k} / \partial k$ , it is unambiguously negative. For any change in capacity utilization, by changing the rate of employment, will cause a change in the same direction in the growth rate of labor supply and, therefore, will cause a change in the opposite direction in the rate of growth of the ratio of capital stock to labor supply.

We now have all the elements for a qualitative phase-diagrammatic analysis of the (local) stability properties of this modified system. As before, we proceed by analyzing the stability of the equilibrium in each one of the three regions in which we divided the relevant domain. The equation describing the  $\hat{k} = 0$  isocline became dependent on the level of  $k$ , though, implying that there is a single non-vertical  $\hat{k} = 0$  isocline in the  $(k - \sigma)$ -space. This makes for the possibility of multiple equilibria, and I turn to this possibility in the next section.

In the preceding specification of the model, the steady-state solution for the LW region ( $\sigma < \sigma^*$ ) would necessarily be a saddle-point, the reason being that  $\text{Det}(M) < 0$  no matter the sign of  $\partial \hat{\sigma} / \partial \sigma$ . In the present specification with an endogenous labor supply growth, that is not necessarily the case anymore. Recall that in this region  $u_{\sigma}^* > 0$  and  $g_{\sigma}^* > 0$ , thus implying that the sign of  $M_{21}^+$  will depend on the relative strength of these two effects. Now, recall that for a higher wage share to raise the growth rate, it has to generate a rise in capacity utilization

which more than compensates the accompanying fall in the share of profits in income, which means that  $u_{\sigma}^* > g_{\sigma}^*$ . Therefore, we will have  $M_{21}^+ < 0$  unless the response of labor supply growth to a change in capacity utilization is low enough to ensure that the growth effect more than compensates the capacity utilization effect on  $\hat{k}$ , despite  $u_{\sigma}^* > g_{\sigma}^*$ . Given the strength of the endogenous labor supply effect, chances for a negative sign for  $M_{21}^+$  are the higher, the greater the extent to which  $u_{\sigma}^*$  is stronger than  $g_{\sigma}^*$ .

Let us then analyze the situation in which  $M_{21}^+ < 0$  and  $M_{11}^+ < 0$ , the latter meaning that the rate of price change is more responsive than the rate of change in nominal wages to changes in capacity utilization. In this case, the economy has a stable long-run equilibrium given that  $\text{Det}(M^+)$  is positive and  $\text{Tr}(M^+)$  is negative. Therefore, while in the preceding model specification the equilibrium solution was not stable in the subset of the domain in which wage-led accumulation ( $g_{\sigma}^* > 0$ ) would obtain, the introduction of endogenous labor supply growth makes for a stable system – provided the rate of change in prices is more responsive than the rate of change in nominal wages to changes in capacity utilization, on the one hand, and the response of the growth rate of labor supply to a change in capacity utilization is not too low and/or the extent to which  $u_{\sigma}^*$  is greater than  $g_{\sigma}^*$  is not too low, on the other hand.

However, the situation may change in case  $M_{11}^+ > 0$ , meaning that the rate of price change is less responsive than the rate of change in nominal wages to changes in capacity utilization, and  $M_{21}^+ < 0$ . The reason is that both the sign of  $\text{Det}(M^+)$  and the sign of  $\text{Tr}(M^+)$  become unclear now, which will require a closer examination of their formal expressions given by

$$\text{Det}(M^+) = au^*[(\mu + \tau + \tau\theta u_{\sigma}^*)\psi - \mu\lambda g_{\sigma}^*] \quad (33)$$

$$\text{Tr}(M^+) = \mu(\lambda a u_{\sigma}^* - 1) - \tau(1 + \theta u_{\sigma}^*) - \psi a u^* \quad (34)$$

Now, recall that for a higher wage share to raise the accumulation rate, it has to generate a rise in capacity utilization which more than offsets the accompanying fall in the profit share, which means that  $u_{\sigma}^* > g_{\sigma}^*$ . Since we are analyzing the situation in which  $M_{11}^+ > 0$ , it follows that  $\text{Det}(M^+) > 0$  and  $\text{Tr}(M^+) < 0$  unless the response of labor supply growth to a change in the rate of employment is too low and/or the response of workers' desired wage share to a change in capacity utilization is too strong. Otherwise, either  $\text{Det}(M^+)$  will be

negative and saddle-point instability will obtain no matter the sign of  $Tr(M^+)$ , or  $Det(M^+)$  will be positive but the sign of  $Tr(M^+)$  will be positive as well and an unstable solution will follow. Hence, a stable equilibrium solution becomes possible even in case the rate of change in nominal wages happens to be more responsive than the rate of change in prices to changes in capacity utilization. In case the response of labor supply growth to a change in the rate of employment is high enough to ensure that  $Det(M^+)$  is positive, but not high enough to ensure that  $Tr(M^+)$  is negative, the resulting equilibrium solution will be unstable. In case the response of labor supply growth to a change in the rate of employment is not high enough to ensure that  $Det(M^+)$  is positive, or the response of the nominal wage growth to a change in capacity utilization is high enough to make for a negative  $Det(M^+)$ , the equilibrium will be saddle-point unstable. In case  $M_{11}^+ < 0$  and  $M_{12}^+ > 0$ , (34) shows that the sign of  $Tr(M^+)$  will be unambiguously negative, which means that whether the equilibrium solution will be stable or saddle-point unstable depends on whether  $Det(M^+)$  will be positive or negative. Now,  $M_{11}^+ < 0$  suggests that the response of the rate of nominal wage change to a change in capacity utilization is weak, while  $M_{21}^+ > 0$  suggests that the response of the rate of growth of labor supply to a change in capacity utilization is weak as well. Given that  $u_\sigma^* > g_\sigma^*$ , (33) shows that  $Det(M^+) > 0$  unless  $\psi$  is much lower than  $\lambda$ . In the event  $\psi$  is sufficiently lower than  $\lambda$  to make for a negative  $Det(M^+)$ , the equilibrium will be saddle-point unstable. Finally, equilibrium will be saddle-point unstable in the event  $M_{11}^+$  and  $M_{21}^+$  are positive, the reason being that now  $Det(M^+)$  will be negative. Hence, a weak endogenous labor supply growth effect coupled with a strong nominal wage growth effect makes for a saddle-point unstable system.

In the IH region ( $\sigma^* < \sigma < \sigma^+$ ), capacity utilization is directly related to the wage share. Changes in the profit share are assumed to dominate changes in capacity utilization, though, thus implying that accumulation is inversely related to the wage share. While a higher wage share will thus put an upward pressure on its own rate of change by raising the wage share desired by workers, it will also raise the markup desired by firms, which will then exert a downward pressure on the rate of change of the wage share. The sign of  $M_{11}^+$  will then depend on the relative strength of these two effects, so that in principle it will be as ambiguous as before. But since the wage share is higher in this region than in the previous one, it is likely that the chances for  $M_{11}^+ > 0$  are higher than in the LW region. As for the sign of

$M_{21}^+$ , it is unambiguously negative, the reason being that  $g_\sigma^*$  is negative in this subset of the relevant domain. In case  $M_{11}^+$  is negative, stability will automatically obtain. In case  $M_{11}^+$  is positive, meaning that the nominal wage change effect is stronger than the price change effect, (33) shows that  $\text{Det}(M^+) > 0$ , and whether stability or instability will obtain depends on the sign of  $\text{Tr}(M^+)$ , which in turn depends on the relative strength of the endogenous labor supply effect with respect to the nominal wage effect. In case the extent to which the nominal wage change effect is greater than the price change effect is larger (smaller) than the response of the rate of change in labor supply to a change in the rate of employment, instability (stability) will obtain. In this subset of the relevant domain, therefore, the introduction of an endogenous mechanism of labor supply growth clearly makes for a more resistant system, in that it takes a nominal wage growth effect stronger than before to cause instability.

In the In the HW region ( $\sigma > \sigma^+$ ), both capacity utilization and accumulation are inversely related to the wage share. The sign of  $M_{21}^+$  is ambiguous, with the reason being as follows. While  $g_\sigma^* < 0$  is making for a negative  $\partial \hat{k} / \partial \sigma$ ,  $u_\sigma^* < 0$  is rather making for a positive one. However,  $g_\sigma^* > u_\sigma^*$  because, for instance, a falling capacity utilization is actually adding to a falling profit share to make for a falling growth rate. Hence, chances for a negative  $M_{21}^+$  are the higher, the weaker the response of the labor supply growth to a change in capacity utilization (and thus employment) and/or the more  $g_\sigma^*$  is greater than  $u_\sigma^*$ . In case  $M_{21}^+$  is negative and  $M_{11}^+ < 0$ , which now means that the rate of change in nominal wages is more responsive than the rate of change in prices to a change in capacity utilization,  $\text{Det}(M^+)$  is positive and  $\text{Tr}(M^+)$  is negative, which in turn makes for a stable equilibrium solution within this subset of the domain.

In case  $M_{21}^+$  is negative but the price change effect is greater than the nominal wage change effect, which makes for  $M_{11}^+ > 0$ , both the sign of  $\text{Det}(M^+)$  and the sign of  $\text{Tr}(M^+)$  become unclear. In this case, (33) and (34) show that chances for  $\text{Det}(M^+) > 0$  are the higher, the greater the extent to which  $g_\sigma^* > u_\sigma^*$ , while chances for  $\text{Det}(M^+) > 0$  and for  $\text{Tr}(M^+) < 0$  together are the higher, the smaller the extent to which the price change effect is greater than the nominal wage change effect and the stronger the endogenous labor supply growth effect. In the event the relative strength of these latter two effects – and of  $u_\sigma^*$  and  $g_\sigma^*$  – is such that  $\text{Det}(M^+) > 0$  but  $\text{Tr}(M^+) > 0$ , an unstable equilibrium will obtain. For

instance, instability would obtain in the event the price change effect is strong enough to dominate the endogenous labor supply effect in the sign of  $\text{Tr}(M^+)$ , even though it is not strong enough to make for a negative  $\text{Det}(M^+)$ . In case the relative strength of all these effects is such that  $\text{Det}(M^+) < 0$ , then a saddle-point unstable equilibrium will obtain. Basically, chances for a negative  $\text{Det}(M^+)$  are the higher, the smaller the extent to which  $g_\sigma^* > u_\sigma^*$ , the greater the extent to which the price change effect is greater than the nominal wage change effect and/or the stronger the endogenous labor supply effect.

As mentioned above, the sign of  $M_{21}^+$  will be negative unless the response of the labor supply growth to a change in capacity utilization (and employment) is very high and/or the extent to which  $g_\sigma^* > u_\sigma^*$  is small enough. Combined with  $M_{11}^+ < 0$ , which in this subset of the domain means that the nominal wage change effect is stronger than the price change effect, a positive sign for  $M_{21}^+$  will make for an ambiguous sign for  $\text{Det}(M^+)$ . The sign of  $\text{Tr}(M^+)$  is negative, though, so that the equilibrium solution will be either stable or saddle-point unstable. Eq. (33) shows that a positive sign for  $\text{Det}(M^+)$  will obtain unless the extent to which  $g_\sigma^* > u_\sigma^*$  is very small and/or the endogenous labor supply effect is significantly stronger than the nominal wage effect. Finally, equilibrium will be unambiguously saddle-point unstable in the event  $M_{21}^+ > 0$ , while the price change effect is stronger than the nominal wage effect, which makes for  $M_{11}^+ > 0$ .

## 5 Multiple equilibria analysis

The non-linearity embodied in the desired accumulation function makes for the possibility of multiple equilibria within the relevant domain, this being the case with either exogenous or endogenous labor supply growth. As shown above, with an exogenously given growth rate of labor supply there may be up to three real values for the share of wages at which a corresponding vertical  $\hat{k} = 0$  isocline would be located in the  $(k - \sigma)$ -space. Hence, it is possible that a configuration with three equilibria obtains within the relevant domain, which would imply path-dependence. More precisely, the equilibrium point towards which, if any at all, the system will converge over time is crucially dependent on the initial conditions. Furthermore, in case the system, having been at some equilibrium point for a while,



is displaced from there due to some exogenous shock, the equilibrium towards which, if any at all, it will converge back depends on what direction of the  $(k - \sigma)$ -space it was displaced to.

Since a broader set of possible multiple equilibria configurations was open by the introduction of an endogenous mechanism of labor supply growth, it is on this case that we will focus more closely in this section. Amongst the possible configurations leading to multiple equilibria, one worthy of a more detailed phase-diagrammatic analysis contains a saddle-point in the LW region, a stable or unstable solution in the IH region and another saddle-point in the HW region. Now, while the sign of  $M_{11}^+ = \partial \hat{\sigma} / \partial \sigma$  depends on the relative bargaining power of capitalists and workers, the sign of  $M_{21}^+ = \partial \hat{k} / \partial \sigma$  depends on the relative response of the accumulation rate and of the growth rate of labor supply to a change in the share of wages. Let us hypothesize that the parameters which govern the relative bargaining power of capitalists and workers can be such that, in response to a change in the wage share, the nominal wage change effect is greater than the price change effect throughout the relevant domain, which implies that  $M_{11}^+ = \partial \hat{\sigma} / \partial \sigma$  is positive in the LW and IH regions and negative in the HW region.

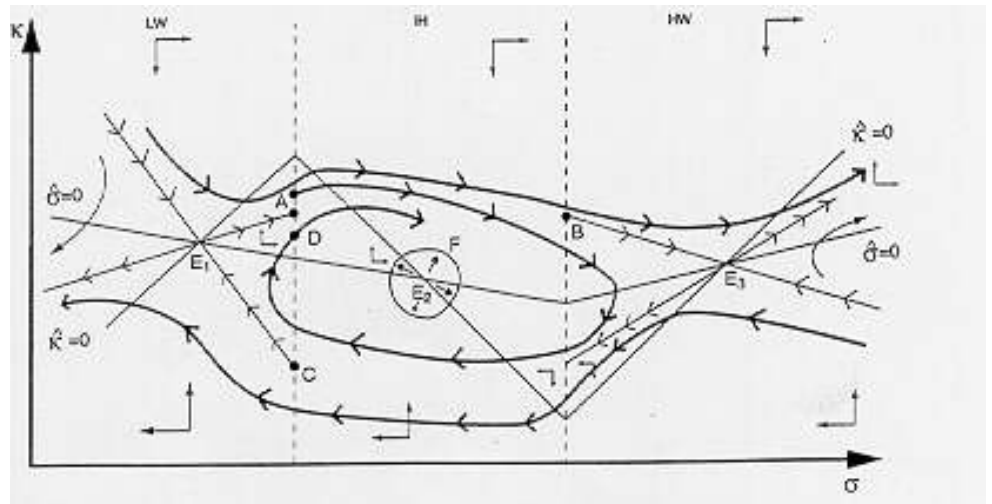
Regarding the sign of  $M_{21}^+ = \partial \hat{k} / \partial \sigma$ , let us hypothesize the following. (31) shows that  $M_{21}^+$  will be positive in the LW region provided  $u_{\sigma}^*$  is not much higher than  $g_{\sigma}^*$  and/or the endogenous labor supply effect is not too strong. In other words,  $M_{21}^+$  will be positive in the event the rate of accumulation is more responsive than the rate of growth of labor supply to a change in the wage share. In the IH region, in turn, it was seen above that  $M_{21}^+$  will be unambiguously negative no matter the relative strength of the several effects in action. Finally, (31) shows that  $M_{21}^+$  will be positive in the HW region provided  $g_{\sigma}^*$  is not much higher than  $u_{\sigma}^*$  and/or the endogenous labor supply effect is not too weak. Recalling that the relative strength of all these effects depends on the levels of  $\sigma$  and  $k$  as well as on parameters of the model, let us hypothesize that those levels and parameters can be such that  $M_{21}^+$  is positive in the LW and HW regions.

Hence, we have a situation in which  $M_{11}^+ = \partial \hat{\sigma} / \partial \sigma$  is positive in the LW and IH regions and negative in the HW region, whereas  $M_{21}^+ = \partial \hat{k} / \partial \sigma$  is positive in the LW and HW regions and negative in the IH region. Recalling that  $M_{12}^+ > 0$  and  $M_{22}^+ < 0$  throughout the domain, a situation like the one being hypothesized

here is pictured in Figure 1. In the LW region, saddle-point instability will obtain, while whether stability or instability will obtain in the IH region depends on the relative strength of  $M_{11}^+ > 0$  and  $M_{22}^+ < 0$ , which will give the sign of  $Tr(M^+)$ , and we analyze these two possible cases in what follows. In the HW region, (33) shows that chances for a negative value for  $Det(M^+)$ , which will make for saddle-point instability, will be the higher, the stronger the endogenous labor supply growth effect, the smaller the extent to which  $g_\sigma^*$  is greater than  $u_\sigma^*$ , and the smaller the extent to which the nominal wage change effect is greater than the price change one. Recalling that the relative strength of all these effects depends on the levels of  $\sigma$  and  $k$  as well as on parameters of the model, let us hypothesize that those levels and parameters can be such that  $Det(M^+)$  is negative in this subset of the relevant domain.

Therefore, an interesting configuration with three equilibria would obtain within the relevant domain, and let us call  $E_1$ ,  $E_2$  and  $E_3$  the equilibrium solutions located in the LW, IH and HW regions, respectively. Having analyzed these three equilibria in isolation in the previous section, we now perform a qualitative phase-diagrammatic analysis of the whole domain. Indeed, we are in position to reveal an interesting potentially cyclical feature of equilibrium points such as  $E_2$ . Suppose we begin a trajectory at point A in Figure 1. The direction of motion of the system indicates that it must flow rightward up until the  $\hat{k} = 0$  isocline is reached, after which the system will flow rightward down. Recall that in the IH region an increase in the wage share will raise capacity utilization but lower the rate of accumulation. Before the  $\hat{k} = 0$  isocline is crossed, though, the net impact of these effects is to raise the level of  $k$ , with the reason being the following. While (31) shows that the rise in capacity utilization and the fall in the rate of accumulation are putting a downward pressure on the level of  $k$ , (32) shows that this fall in the level of  $k$  will raise the rate of growth of  $k$  more than proportionately to make for a resulting rise in the level of  $k$ . Once the  $\hat{k} = 0$  isocline is reached, though, further increases in the wage share will lead to a fall in the level of  $k$ .

Figure 1



Now, recall that the fixed-proportion nature of the production technology implies that capacity utilization and the rate of employment move in the same direction, with the strength of this connection being given by  $ak$ . In other words, changes in  $k$  lead to changes in the relative impact of higher capacity utilization on the share of wages desired by workers and on the markup desired by firms. Besides, what makes a rise in the wage share even more self-undermining is that by raising capacity utilization and thus employment, it raises the rate of growth of labor supply. Since we have hypothesized above that in response to a change in the wage share the nominal wage change effect is stronger than the price change effect, though, the system will keep flowing rightward down until it reaches either the HW region or the  $\hat{\sigma} = 0$  isocline in the same IH region.

Suppose the system reaches the HW region before reaching the  $\hat{\sigma} = 0$  isocline. In case it reaches that region through anywhere above point B, the system will keep flowing rightward down until it reaches the  $\hat{k} = 0$  isocline, after which it will flow rightward up and further away from  $E_3$  – and actually from anyone of the other equilibria. Indeed, it is only by a happy fluke that the system will enter the HW region through point B, which would take it to  $E_3$ . Suppose the system reaches that region through some point below B. Once inside the HW region, it will keep flowing rightward down until the  $\hat{\sigma} = 0$  isocline is reached, where the motion of the system will then undergo an important qualitative change. Recall that in this

subset of the domain capacity utilization and accumulation are both inversely related to the wage share. Before the  $\hat{\sigma} = 0$  isocline is reached, an increase in the wage share, by lowering the rate of accumulation by more than it lowers the growth rate of labor supply, will lower the level of  $k$ . However, it is only when the system reaches the  $\hat{\sigma} = 0$  isocline that  $\sigma$  is high enough – and  $k$  is low enough – for a further increase in the wage share to have a negative impact on its rate of growth that more than offsets that increase. With the wage (profit) share now falling (rising), capacity utilization and accumulation both start rising again, even though  $k$  keeps falling, the reason being that the rise in capacity utilization raises the growth rate of labor supply by more than the rise in the accumulation rate.

The system will flow leftward down until it reaches back the IH region, after which it will keep such motion for a while. Though capacity utilization is falling and profit-led accumulation prevails, the levels of  $\sigma$  and  $k$  have not fallen enough yet to ensure the reversal of the leftward down motion of the system. Once it reaches the  $\hat{k} = 0$  isocline, though, a decrease in the wage share, despite leading to a further decrease in it, will now raise the level of  $k$ , so that the system will start moving leftward up. A happy fluke may take the system to point C, which will then lead it to converge to  $E_1$ . In the event the system reaches the LW region through a point below C, it will flow leftward up and then, having crossed the  $\hat{k} = 0$  isocline, leftward down, with falling levels of both  $\sigma$  and  $k$ . Given the saddle-point nature of  $E_1$ , the level of  $k$  could not rise to the extent that was necessary to allow the wage share to start increasing again. Let us suppose the system enters the LW region through a point above C, and recall that both capacity utilization and accumulation are wage-led in that region. Since the former is dominating the latter in the growth rate of  $\hat{k}$ , the level of  $k$  can keep rising and thus reach the  $\hat{\sigma} = 0$  isocline, after which the wage share will start rising again. Besides, since the capacity utilization effect is still being dominated by the accumulation effect in  $\hat{k}$ , the level of  $k$  keeps rising. Once the system reaches back the IH region – through, say, point D – the cyclical motion just described will re-start. However, this inner part of the cyclical trajectory will not cross the previous one, since trajectories of differential equations with continuous partial derivatives must be unique (Arrowsmith & Place, 1992). Furthermore, this inner part may or may not pass through the HW and LW regions again, it being possible that the two changes of sign of  $\hat{\sigma}$  occur within in the IH region itself. Now, recall that whether  $E_2$  is stable or unstable depends on the relative strength of the positive  $\partial \hat{\sigma} / \partial \sigma$  with respect

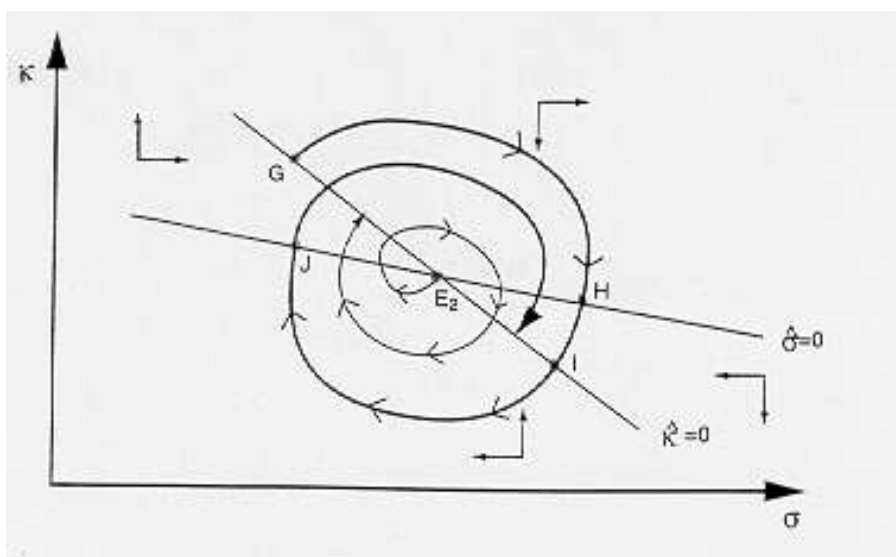
the negative  $\partial \hat{k} / \partial k$ . More precisely,  $E_2$  will be locally stable (unstable) in case the response of the rate of growth of labor supply to a change in the level of  $k$  is stronger (weaker), in absolute value, than the extent to which the response of the rate of nominal wage change is stronger than the rate of price change to a change in the level of  $\sigma$ .

In the event  $E_2$  is locally stable, there is a region around it in which all trajectories tend to  $E_2$ . Since the system will end up reaching that region along the trajectory started at A, it will converge to  $E_2$ . In case  $E_2$  is unstable, which is the situation pictured in Figure 1, there is a neighborhood of  $E_2$ , say F, in which all trajectories of the system will move away from  $E_2$ . Since the system will end up reaching that neighborhood along the trajectory initiated at point A, it will not reach  $E_2$ . Indeed, there may eventually be a closed, bounded area encircling the neighborhood F and from which no trajectory will exit. In case this area contains no equilibrium points, the Poincaré-Bendixson theorem would ensure that it must then contain at least one stable limit cycle: any point in the area not on a limit cycle would be attracted to such a cycle (Arrowsmith & Place, 1992). Whether or not some limit cycle will emerge, the system will move cyclically in the IH region, which shows its propensity to experience endogenous, self-sustaining fluctuations in the capital to labor supply ratio and distributive shares, with capacity utilization, accumulation and employment rate fluctuating as well.

Suppose, for instance, that the economy starts from point G in Figure 2. A rise in the wage share will raise capacity utilization and lower the accumulation rate, which will then put an unambiguously downward pressure on the level of  $k$ . Since the response of the rate of change in nominal wages is stronger than the response of the rate of price change to a change in capacity utilization, this rise in the wage share will engender a further increase of it. Now, this fall in  $k$  will put a downward pressure on the wage share by lowering the employment rate for a given degree of capacity utilization. But this fall in  $k$  will also put an upward pressure on itself because the fall in the employment rate will lower the growth of labor supply for a given rate of accumulation. Nonetheless, the level of  $k$  is still high enough and the level of  $\sigma$  still low enough to ensure that a fall in the former and a rise in the latter follows. As the system approaches point H, though, the extent to which the nominal wage effect is stronger than the price change effect in  $\hat{\sigma}$  is falling.

Once point H is reached, the wage share (capital to labor supply ratio) will have risen (fallen) by enough to make for a stationary wage share. However,  $k$  is still falling, the reason being that it has not fallen enough to engender a reversal of such downward trend. With the wage (profit) share falling (rising) now, a falling capacity utilization and a rising rate of accumulation are putting an upward pressure on  $k$ , while a rise in  $k$ , by raising the employment rate for a given capacity utilization, will put a downward pressure on  $k$  by raising the growth of labor supply for a given accumulation rate. Once the system reaches point I, though,  $k$  and  $\sigma$  will have fallen by enough for  $k$  to be stationary. But since the wage share is falling,  $k$  will soon start rising, the reason being that at those levels of  $\sigma$  and  $k$  the accumulation effect will soon become stronger than the labor supply effect in  $\hat{k}$ . As the system approaches point J, though, the extent to which the nominal wage effect is stronger than the price change effect in  $\hat{\sigma}$  is rising. Once point J is reached, the values of the two state variables will be such that the downward trend in the wage share will cease. At this point the cyclical motion just described will re-start.

Figure 2



Hence, this model shares with the classic contribution by Goodwin (1967) a cyclical growth dynamics governed by the interaction between accumulation of

capital, distribution and employment.<sup>14</sup> Unlike the Goodwin model, though, this one allows effective demand to play an active role through a variable degree of capacity utilization, incorporates price and nominal wage dynamics through a conflict theory of inflation, and introduces a feedback from the dynamics of the labor and goods market to the growth of labor supply. If left undisturbed, the Goodwin model will produce conservative cyclical fluctuations in distributive shares and in the rate of employment. However, the trajectories will no longer be closed orbits if direct feedbacks from the wage share to its rate of growth – or from the level of the employment rate to its rate of growth – are introduced. The model of this essay introduces both feedbacks via variable capacity utilization, conflict inflation and endogenous labor supply growth. Indeed, the first two features are shared with Dutt (1992), from which this model draws the most. Unlike the latter, though, this model does not rely on full capacity being reached for profit-led accumulation and multiple equilibria to obtain. Given the non-linear investment function used here, the system may experience self-sustaining fluctuations in the state variables – eventually alternating phases of wage-led accumulation with phases of profit-led accumulation – below full capacity utilization. Indeed, the non-linear nature of the model allows it to specify precise ranges of distributive shares within which wage-led and profit-led capacity utilization and accumulation obtain.

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(14) In the Goodwin model, the share of wages is determined by the reserve army of labor, or, more precisely, by the employment rate. The pace of capital accumulation determines the demand for labor. If the rate of accumulation is rising sufficiently, so does the employment rate. Beyond a certain value of the employment rate (i.e. in the neighborhood of full employment) the vigorous accumulation of capital leads to a rising real wage and a rising wage share. This process goes on until the rise in the wage share is sufficient to reduce the rate of profit to a point where the rate of accumulation slows down and unemployment begin to rise. The replenishment of the reserve army of labor yields a falling wage share, therefore a rising rate of profit and eventually an upturn in the rate of accumulation.

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